

Name: _____

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A-Level Mathematics — Practice Exam

Instructions: Answer all questions in the spaces provided. Show your working — method marks are awarded even when the final answer is wrong. The answer key with worked solutions is available at examtex.com/free-exams.

Time Limit: 75 minutes

Calculations (8 questions)

[4 points] 1. Indices and surds.

[2 points] (a) Simplify $\frac{8^{\frac{2}{3}} \times 4^{-\frac{1}{2}}}{2^{-3}}$, giving your answer as an integer.

[2 points] (b) Rationalise the denominator of $\frac{14}{3-\sqrt{2}}$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers.

[5 points] 2. The function f is defined by $f(x) = 2x^2 - 12x + 23$.

$$f(x) = 2x^2 - 12x + 23$$

[2 points] (a) Express $f(x)$ in the form $a(x + b)^2 + c$, where a , b and c are integers to be found.

[1 point] (b) Hence write down the coordinates of the vertex of the curve $y = f(x)$.

[2 points] (c) Using the discriminant, determine the number of real roots of the equation $f(x) = 0$.

[6 points] 3. The circle C has centre $(3, -2)$ and passes through the point $P(7, 1)$.

[3 points] (a) Find an equation for C .

[3 points] (b) Find an equation of the tangent to C at the point P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

[5 points] 4. This question is about the binomial expansion of $(2 + 3x)^5$.

$$(2 + 3x)^5$$

- [3 points]** (a) Find the first four terms, in ascending powers of x , of the binomial expansion of $(2 + 3x)^5$.
Give each coefficient as an integer.

- [2 points]** (b) Use your expansion with a suitable value of x to estimate 2.03^5 , giving your answer to 4 decimal places.

- [7 points]** 5. A curve has equation $y = 2x^3 - 9x^2 + 12x - 3$.
 $y = 2x^3 - 9x^2 + 12x - 3$

- [4 points]** (a) Find the coordinates of the stationary points of the curve.

- [2 points]** (b) Determine the nature of each stationary point.

[1 point] (c) State the range of values of x for which y is increasing.

[6 points] 6. The curve with equation $y = 4x - x^2$ and the line with equation $y = x$ enclose a finite region R . (For a sketch: the curve is an inverted parabola through the origin with maximum at $(2, 4)$; the line passes through the origin with gradient 1 and cuts the curve again to the right, with the curve above the line between the two intersections.)

$$y = 4x - x^2, \quad y = x$$

[2 points] (a) Find the coordinates of the points of intersection of the curve and the line.

[4 points] (b) Find the exact area of the region R .

[6 points] 7. Solve the equation $2 \sin^2 \theta + 3 \cos \theta = 3$ for $0^\circ \leq \theta \leq 360^\circ$. Show each step of your working.

$$2 \sin^2 \theta + 3 \cos \theta = 3, \quad 0^\circ \leq \theta \leq 360^\circ$$

[8 points] 8. Statistics. The discrete random variable X has the probability distribution $P(X = 1) = 0.1$, $P(X = 2) = 0.3$, $P(X = 3) = k$, $P(X = 4) = 0.2$, where k is a constant.

$$P(X = 1) = 0.1, \quad P(X = 2) = 0.3, \quad P(X = 3) = k, \quad P(X = 4) = 0.2$$

[1 point] (a) Find the value of k .

[4 points] (b) Find $E(X)$ and $\text{Var}(X)$.

- [3 points]** (c) Five independent observations of X are taken. Find the probability that exactly two of them take the value 4.

Proofs (1 question)

- [4 points]** 9. Prove that, for every integer n , $(2n + 1)^2 - 1$ is divisible by 8.