

Name: _____

FREE PRACTICE EXAM · EXAMTEX.COM/FREE-EXAMS

AP Calculus AB — Practice Exam

Instructions: No calculator permitted. 16 multiple-choice questions (four answer choices each, 1 point each) followed by one free-response question (9 points), 45 minutes in total. Wrong multiple-choice answers are not penalized. On the free-response question, show all your work and justify your answers; exact, unsimplified answers are acceptable.

Time Limit: 45 minutes

Multiple Choice (16 questions)

- [1 point] 1. The table gives selected values of the differentiable functions f and g and their derivatives. If $h(x) = f(g(x))$, what is the value of $h'(2)$?

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	-4	0	3
2	3	-3	4	5
3	1	-2	7	2
4	-1	-2	8	0

- A. -15
- B. -2
- C. -10
- D. -6

- [1 point] 2. The graph of the function g on the interval $0 \leq x \leq 6$ consists of two line segments: one from $(0, 1)$ to $(3, 7)$ and one from $(3, 7)$ to $(6, 4)$. If $h(x) = g(x^2)$, what is the value of $h'(2)$?

- A. -4
- B. -1
- C. 8
- D. 24

- [1 point] 3. If $y = \cos^3(2x)$, then $\frac{dy}{dx} =$

- A. $3 \cos^2(2x)$
- B. $-3 \cos^2(2x) \sin(2x)$
- C. $6 \cos^2(2x) \sin(2x)$
- D. $-6 \cos^2(2x) \sin(2x)$

[1 point] 4. If $f(x) = \sqrt{2x^2 + 1}$, what is the slope of the line tangent to the graph of f at $x = 2$?

- A. $\frac{1}{6}$
- B. $\frac{4}{3}$
- C. $\frac{8}{5}$
- D. $\frac{2}{3}$

[1 point] 5. The curve defined by $x^2y + y^3 = 10$ passes through the point $(3, 1)$. What is the value of $\frac{dy}{dx}$ at $(3, 1)$?

- A. -2
- B. $-\frac{1}{2}$
- C. -1
- D. 0

[1 point] 6. At which of the following points on the curve $x^2 + 2xy + 3y^2 = 6$ is the line tangent to the curve horizontal?

- A. $(0, \sqrt{2})$
- B. $(3, -1)$
- C. $(1, -1)$
- D. $(\sqrt{3}, -\sqrt{3})$

[1 point] 7. Which of the following is an equation of the line tangent to the curve $xe^y + y = 2$ at the point $(2, 0)$?

- A. $y = -\frac{1}{3}(x - 2)$
- B. $y = -\frac{1}{2}(x - 2)$
- C. $y = -3(x - 2)$
- D. $y = 3(x - 2)$

[1 point] 8. Let $f(x) = x^3 + 2x + 1$, and let g be the inverse function of f . What is the value of $g'(4)$?

- A. $\frac{1}{50}$
- B. 5
- C. $\frac{1}{5}$
- D. 1

[1 point] 9. The function f is differentiable and increasing, and selected values of f and its derivative f' are given in the table. If g is the inverse function of f , what is the value of $g'(5)$?

x	1	2	3	5
$f(x)$	0	2	5	15
$f'(x)$	1	3	4	6

- A. $\frac{1}{4}$
- B. $\frac{1}{6}$
- C. 3
- D. 4

[1 point] 10. The function f is differentiable and one-to-one, with $f(2) = 7$ and $f'(2) = -3$. Which of the following is an equation of the line tangent to the graph of f^{-1} at $x = 7$?

- A. $y - 7 = -\frac{1}{3}(x - 2)$
- B. $y - 2 = -3(x - 7)$
- C. $y - 2 = \frac{1}{3}(x - 7)$
- D. $y - 2 = -\frac{1}{3}(x - 7)$

[1 point] 11. $\frac{d}{dx} [\arctan(3x)] =$

- A. $\frac{1}{1 + 9x^2}$
- B. $\frac{3}{1 + 9x^2}$
- C. $\frac{3}{1 + 3x^2}$
- D. $3 \sec^2(3x)$

[1 point] 12. If $f(x) = \arccos(2x)$, then $f'(\frac{1}{4}) =$

- A. $\frac{4\sqrt{3}}{3}$
- B. $-\frac{2\sqrt{3}}{3}$
- C. $-\frac{4\sqrt{3}}{3}$
- D. $-\frac{8\sqrt{15}}{15}$

[1 point] 13. If $f(x) = \ln\left(\frac{x^2}{x+1}\right)$ for $x > 0$, then $f'(1) =$

- A. 2
- B. $\frac{3}{4}$
- C. $\frac{5}{2}$
- D. $\frac{3}{2}$

[1 point] 14. If $h(x) = x \arcsin x$, then $h'(\frac{1}{2}) =$

- A. $\frac{\pi}{6} + \frac{\sqrt{3}}{3}$
- B. $\frac{2\sqrt{3}}{3}$
- C. $\frac{\pi}{6} + \frac{2\sqrt{3}}{3}$
- D. $\frac{\pi}{3} + \frac{\sqrt{3}}{3}$

[1 point] 15. For the curve $x^2 - y^2 = 16$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(5, 3)$?

- A. $\frac{1}{3}$
- B. $\frac{16}{27}$
- C. $-\frac{16}{27}$
- D. $-\frac{16}{9}$

[1 point] 16. If $f(x) = \ln(1 + x^2)$, then $f''(2) =$

- A. $\frac{6}{25}$
- B. $-\frac{6}{25}$
- C. $\frac{2}{5}$
- D. $-\frac{6}{5}$

Calculations (1 question)

[9 points] 17. Implicit differentiation. Consider the curve given by the equation $x^3 - 2xy + y^2 = 4$. The point $(1, 3)$ lies on the curve.

$$x^3 - 2xy + y^2 = 4$$

[2 points] (a) Show that $\frac{dy}{dx} = \frac{3x^2 - 2y}{2x - 2y}$.

[2 points] (b) Write an equation for the line tangent to the curve at the point $(1, 3)$.

[3 points] (c) Find the coordinates of every point on the curve at which the line tangent to the curve is vertical. Justify your answer.

[2 points] (d) Evaluate $\frac{d^2y}{dx^2}$ at the point $(1, 3)$.