Name: _____

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AP Calculus AB — Practice Exam

Instructions: No calculator permitted. 16 multiple-choice questions (four answer choices each, 1 point each) followed by one free-response question (9 points), 45 minutes in total. Wrong multiple-choice answers are not penalized. On the free-response question, show all your work, justify your answers and include units of measure where a part asks for them; exact, unsimplified answers are acceptable.

Time Limit: 45 minutes

Multiple Choice (16 questions)

[1 point] 1. A hiker walks along a mountain trail. Her elevation, in meters above sea level, after she has walked d kilometers along the trail is $E(d)$, where E is differentiable. Which of the following is the best interpretation of $E'(3) = -40$?

A. After walking 3 kilometers, the hiker is 40 meters lower than her starting elevation.

B. When the hiker has walked 3 kilometers, her elevation is decreasing at a rate of 40 meters per kilometer walked.

C. Over the first 3 kilometers, her elevation decreases at an average rate of 40 meters per kilometer.

D. When the hiker has walked 3 kilometers, she is descending at a rate of 40 meters per hour.

[1 point] 2. The height of a sunflower, in centimeters, t days after it is planted is given by the increasing differentiable function $H(t)$. Let g be the inverse function of H, so that $g(y)$ is the number of days it takes the sunflower to reach a height of y centimeters. Which of the following is the best interpretation of $g'(30) = 2$?

A. On day 30 after planting, the sunflower is growing at a rate of 2 centimeters per day.

B. When the sunflower is 30 centimeters tall, it is growing at a rate of 2 centimeters per day.

C. The sunflower reaches a height of 30 centimeters 2 days after it is planted.

D. When the sunflower is 30 centimeters tall, it is growing at a rate of $\frac{1}{2}$ centimeter per day.

[1 point] 3. A particle moves along the x-axis so that its position at time $t > 0$ is $x(t) = 2t^3 - 21t^2 + 60t$. For which values of t is the speed of the particle increasing?

A. $t > 3.5$ only
B. $t > 5$ only
C. $2 < t < 3.5$ and $t > 5$
D. $0 < t < 2$ and $3.5 < t < 5$

[1 point] 4. An object attached to a spring moves along a vertical line so that its position, in centimeters, at time t seconds is $y(t) = 6\cos\left(\frac{\pi t}{4}\right)$. What is the acceleration of the object at the first time $t > 0$ at which the object is at rest?

A. $\frac{3\pi^2}{8}$ centimeters per second per second
B. $-\frac{3\pi^2}{8}$ centimeters per second per second
C. 0 centimeters per second per second
D. $\frac{3\pi}{2}$ centimeters per second per second

[1 point] 5. A ball is thrown upward from the roof of a building. Its height above the ground, in feet, t seconds after it is thrown is $h(t) = -16t^2 + 64t + 80$. What is the velocity of the ball at the instant it hits the ground?

A. 96 feet per second
B. -64 feet per second
C. -96 feet per second
D. -160 feet per second

[1 point] 6. The amount of a medication in a patient's bloodstream t hours after an injection is modeled by $A(t) = 50te^{-t/2}$ milligrams. At what rate is the amount of medication in the bloodstream changing at time $t = 4$ hours?

A. $\frac{200}{e^2}$ milligrams per hour
B. $-\frac{50}{e^2}$ milligrams per hour
C. $-\frac{150}{e^2}$ milligrams per hour
D. $-\frac{25}{e^2}$ milligrams per hour

[1 point] 7. A metal rod is heated at one end. The temperature of the rod, in degrees Celsius, at a distance of x centimeters from the heated end is $T(x)$, where T is differentiable. Selected values of $T(x)$ are given in the table. Using the data in the table, what is the best estimate of $T'(3.5)$?

x	0	2	5	9
$T(x)$	84	75	63	55

A. -4 degrees Celsius per centimeter
B. -12 degrees Celsius per centimeter
C. $-\frac{29}{9}$ degrees Celsius per centimeter
D. -0.25 centimeter per degree Celsius

[1 point] 8. The variables x and y are differentiable functions of t that satisfy $xy^2 = 12$ for all t. At the instant when $x = 3$ and $y = 2$, $\frac{dx}{dt} = 4$. What is the value of $\frac{dy}{dt}$ at that instant?

A. $-\frac{1}{3}$
B. $-\frac{8}{3}$
C. 0
D. $-\frac{4}{3}$

[1 point] 9. The radius r and the height h of a right circular cylinder are both changing with time t. The volume of the cylinder is $V = \pi r^2 h$. Which of the following is an expression for $\frac{dV}{dt}$?

A. $\pi\left(2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right)$
B. $2\pi rh\frac{dr}{dt}$
C. $2\pi r\frac{dr}{dt} \cdot \frac{dh}{dt}$
D. $\pi r^2\frac{dh}{dt}$

[1 point] 10. Car A is 9 kilometers east of an intersection and is driving west, toward the intersection, at 50 kilometers per hour. At the same moment, car B is 12 kilometers north of the intersection and is driving north, away from the intersection, at 30 kilometers per hour. At that moment, how is the straight-line distance between the two cars changing?

A. It is increasing at 54 kilometers per hour.
B. It is decreasing at 20 kilometers per hour.
C. It is decreasing at 6 kilometers per hour.
D. It is decreasing at 90 kilometers per hour.

[1 point] 11. Water drains from a tank shaped like an inverted right circular cone, vertex at the bottom, with a radius of 3 meters at the top and a height of 6 meters. The water drains at a constant rate of 2 cubic meters per minute. At what rate is the depth of the water changing at the instant the water is 4 meters deep? (The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.)

A. $-\frac{3}{2\pi}$ meters per minute
B. $-\frac{1}{2\pi}$ meter per minute
C. $-\frac{1}{6\pi}$ meter per minute
D. $-\frac{1}{32\pi}$ meter per minute

[1 point] 12. Let $f(x) = \ln(2x - 5)$. What is the approximation of $f(3.04)$ given by the line tangent to the graph of f at $x = 3$?

A. 0.04
B. 1.08
C. 6.08
D. 0.08

[1 point] 13. The function f is differentiable, with $f(3) = 4$ and $f'(x) = \sqrt{x^2 + 7}$ for all x. The line tangent to the graph of f at $x = 3$ is used to approximate $f(2.9)$. What is the approximation, and is it an overestimate or an underestimate of $f(2.9)$?

A. 3.6, an overestimate
B. 3.6, an underestimate
C. 4.4, an underestimate
D. 4.4, an overestimate

[1 point] 14. $\lim_{x \to 0} \dfrac{1 - \cos(3x)}{x^2}$ is

A. $\frac{9}{2}$
B. $\frac{3}{2}$
C. 9
D. $\frac{1}{2}$

[1 point] 15. Let f be a twice-differentiable function with $f(1) = 2$ and $f'(1) = 6$. What is the value of $\lim_{x \to 1} \dfrac{f(x) - 2}{\sin(\pi x)}$?

A. $\frac{6}{\pi}$
B. -6
C. $-\frac{\pi}{6}$
D. $-\frac{6}{\pi}$

[1 point] 16. $\lim_{x \to \infty} \dfrac{\ln\left(1 + e^{2x}\right)}{5x}$ is

A. 0
B. $\frac{1}{5}$
C. $\frac{2}{5}$
D. ∞

Calculations (1 question)

[9 points] 17. Oil slick. After a leak from a damaged ship is stopped, the oil on the surface of the sea spreads out as a circular slick. The radius of the slick, in meters, is modeled by a twice-differentiable, increasing function r of time t, measured in hours since the leak was stopped, for $0 \leq t \leq 8$. Selected values of $r(t)$ and of its derivative $r'(t)$ are given in the table.

t (hours)	0	2	4	6	8
$r(t)$ (meters)	10	18	24	28	30
$r'(t)$ (meters per hour)	4.5	3.5	2.5	1.5	0.5

[2 points] (a) Use the data in the table to estimate $r''(5)$. Show the computation that leads to your answer, and explain the meaning of your answer in the context of the problem, using correct units.

[3 points] (b) Write an equation for the line tangent to the graph of r at $t = 6$, and use it to approximate the radius of the slick at time $t = 6.4$ hours. Given that $r''(t) < 0$ for $0 < t < 8$, is your approximation an overestimate or an underestimate of $r(6.4)$? Give a reason for your answer.

[2 points] (c) Find the rate at which the area of the slick is changing at time $t = 4$ hours. Indicate units of measure.

[2 points] (d) The oil has the same thickness w, in meters, everywhere in the slick, so the volume of oil in the slick is $V = \pi r^2 w$ cubic meters. No oil is added or removed after $t = 0$, so V is constant. At time $t = 4$ hours, the thickness of the slick is 0.0048 meter. Find the rate at which the thickness of the slick is changing at time $t = 4$. Indicate units of measure.

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