Name: _____

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AP Calculus AB — Practice Exam

Instructions: No calculator permitted. 16 multiple-choice questions (four answer choices each, 1 point each) followed by one free-response question (9 points), 45 minutes in total. Wrong multiple-choice answers are not penalized. On the free-response question, show all your work and justify your answers; exact, unsimplified answers are acceptable.

Time Limit: 45 minutes

Multiple Choice (16 questions)

[1 point] 1. Let $f(x) = x^3 - 3x + 1$. Which value or values of c satisfy the conclusion of the Mean Value Theorem for f on the interval $0 \le x \le 3$?

A. $c = 1$ only
B. $c = \sqrt{2}$ only
C. $c = \frac{3}{2}$ only
D. $c = \sqrt{3}$ only

[1 point] 2. The function h is differentiable for all real numbers. Selected values of h are given in the table. Which of the following must be true?

x	0	2	5	7
$h(x)$	4	10	1	8

A. $h'(c) = -9$ for some c with $2 < c < 5$
B. $h'(x) < 0$ for all x with $2 < x < 5$
C. $h'(c) = -3$ for some c with $2 < c < 5$
D. $h(c) = 0$ for some c with $0 < c < 7$

[1 point] 3. For which of the following functions does the Extreme Value Theorem guarantee that the function has both an absolute maximum value and an absolute minimum value on the given interval?

A. $f(x) = |x - 1|$ on $0 \le x \le 3$
B. $f(x) = \dfrac{1}{x - 1}$ on $0 \le x \le 2$
C. $f(x) = x^2$ on $0 < x < 2$
D. $f(x) = \begin{cases} x + 1, & 0 \le x < 2 \\ 0, & 2 \le x \le 4 \end{cases}$ on $0 \le x \le 4$

[1 point] 4. Let $f(x) = \dfrac{x^2}{x+2}$. What are the x-coordinates of all the critical points of f?

A. $x = 0$ only
B. $x = -2$ and $x = 0$
C. $x = -4$ and $x = 0$
D. $x = -4$, $x = -2$ and $x = 0$

[1 point] 5. Let $f(x) = (x^2 - 3)e^x$. For which values of x is f increasing?

A. $-\sqrt{3} < x < \sqrt{3}$
B. $-3 < x < 1$
C. $x < -\sqrt{3}$ or $x > \sqrt{3}$
D. $x < -3$ or $x > 1$

[1 point] 6. The derivative of a function f is $f'(x) = x^3(x-2)^2(x+5)$. At which values of x does f have a relative minimum?

A. $x = 0$ only
B. $x = -5$ only
C. $x = 0$ and $x = 2$
D. $x = -5$ and $x = 2$

[1 point] 7. Let $f(x) = x + 2\cos x$. On the interval $0 < x < 2\pi$, at which value of x does f have a relative minimum?

A. $x = \frac{\pi}{6}$
B. $x = \frac{5\pi}{6}$
C. $x = \frac{3\pi}{2}$
D. $x = \frac{11\pi}{6}$

[1 point] 8. Let $g(x) = 2x^3 - 3x^2 - 12x + 1$. What is the absolute minimum value of g on the closed interval $-3 \leq x \leq 3$?

A. -44
B. -19
C. -8
D. 8

[1 point] 9. Let $f(x) = x^2 \ln x$ for $x > 0$. On which interval is the graph of f concave up?

A. $0 < x < e^{-3/2}$
B. $0 < x < e^{-1/2}$
C. $x > e^{-3/2}$
D. $x > e^{-1/2}$

[1 point] 10. The function $f(x) = x^3 + ax^2 - 9x + 4$, where a is a constant, has a critical point at $x = 3$. What is the value of a, and what does the second derivative test show about f at $x = 3$?

A. $a = -3$, and f has a relative maximum at $x = 3$
B. $a = -3$, and f has a relative minimum at $x = 3$
C. $a = 3$, and f has a relative maximum at $x = 3$
D. $a = 3$, and f has a relative minimum at $x = 3$

[1 point] 11. The function f is differentiable on the closed interval $-4 \leq x \leq 4$. The graph of its derivative f' consists of three line segments joining, in order, the points $(-4, 2)$, $(-2, -2)$, $(1, 4)$ and $(4, -2)$. On which open interval is the graph of f both decreasing and concave up?

A. $-3 < x < -2$
B. $-3 < x < -1$
C. $-2 < x < 1$
D. $-2 < x < -1$

[1 point] 12. A function f is twice differentiable for all real numbers. The graph of f has a relative minimum at $x = -2$, a relative maximum at $x = 4$, no other relative extrema, and exactly one point of inflection, at $x = 1$. Which of the following could be $f'(x)$?

A. $(x + 2)(4 - x)$
B. $(x + 2)(x - 4)$
C. $(x - 1)^2$
D. $2 - 2x$

[1 point] 13. The function g is twice differentiable on the interval $-1 < x < 4$. The table gives the sign or the value of g' and g'' at $x = 0$, at $x = 2$ and on the intervals between them. Which of the following is true?

x	$-1 < x < 0$	0	$0 < x < 2$	2	$2 < x < 4$
$g'(x)$	negative	0	positive	positive	positive
$g''(x)$	positive	positive	positive	0	negative

A. g has a relative maximum at $x = 0$ and a point of inflection at $x = 2$.
B. g has a relative minimum at $x = 0$ and a relative maximum at $x = 2$.
C. g has a relative minimum at $x = 0$ and a point of inflection at $x = 2$.
D. g has a relative maximum at $x = 0$ and a relative maximum at $x = 2$.

[1 point] 14. A rectangle has its lower side on the x-axis and its two upper vertices on the parabola $y = 12 - x^2$. Let $x > 0$ be the x-coordinate of the upper-right vertex. Which of the following gives the area A of the rectangle as a function of x, together with the interval of x-values on which A should be maximized?

A. $A(x) = x(12 - x^2)$ for $0 < x < 2\sqrt{3}$
B. $A(x) = 2x(12 - x^2)$ for $0 < x < 2\sqrt{3}$
C. $A(x) = x(12 - x^2)$ for $0 < x < 12$
D. $A(x) = 2x(12 - x^2)$ for $0 < x < 12$

[1 point] 15. A farmer has 240 meters of fencing to enclose a rectangular field that borders a straight river. No fencing is needed along the river. What is the largest possible area of the field?

A. 6400 square meters
B. 7200 square meters
C. 10800 square meters
D. 14400 square meters

[1 point] 16. The points $(1, 2)$ and $(1, -2)$ lie on the curve $y^2 = x^3 - 3x + 6$, and $\dfrac{dy}{dx} = 0$ at both points. Which of the following is true?

A. y has a relative maximum at $(1, 2)$ and a relative minimum at $(1, -2)$.
B. y has a relative minimum at both $(1, 2)$ and $(1, -2)$.
C. Neither point is a relative extremum, because the curve is not the graph of a function.
D. y has a relative minimum at $(1, 2)$ and a relative maximum at $(1, -2)$.

Calculations (1 question)

[9 points] 17. Curve analysis. Let f be the function defined by $f(x) = (x - 4)\sqrt[3]{x}$ for $-1 \le x \le 8$, where $\sqrt[3]{x}$ is the real cube root of x, defined for every real number (so $\sqrt[3]{-1} = -1$).

$$f(x) = (x - 4)\sqrt[3]{x}, \qquad -1 \le x \le 8$$

[3 points] (a) Find $f'(x)$. Then find the x-coordinate of each critical point of f on the open interval $-1 < x < 8$, and classify each one as the location of a relative minimum, a relative maximum, or neither. Justify your answers.

[2 points] (b) Find the absolute maximum value and the absolute minimum value of f on the closed interval $-1 \leq x \leq 8$. Justify your answers.

[2 points] (c) On which open intervals in $-1 < x < 8$ is the graph of f concave up, and on which is it concave down? Find the x-coordinate of each point of inflection of the graph of f on $-1 < x < 8$, or explain why there is none. Justify your answers.

[2 points] (d) Find the average rate of change of f over the interval $-1 \leq x \leq 8$. Does the Mean Value Theorem guarantee a value c, with $-1 < c < 8$, such that $f'(c)$ equals this average rate of change? Justify your answer.

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