

Name: _____

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AP Calculus BC — Practice Exam

Instructions: No calculator permitted. Show all working; exact, unsimplified answers are acceptable.

Time Limit: 60 minutes

Multiple Choice (12 questions)

[2 points] 1. Evaluate $\lim_{x \rightarrow 0} \frac{e^{2x} - 1 - 2x}{x^2}$.

- A. 0
- B. 1
- C. 2
- D. 4

[2 points] 2. $\int x e^{3x} dx =$

- A. $\frac{e^{3x}}{9}(3x - 1) + C$
- B. $\frac{e^{3x}}{9}(3x + 1) + C$
- C. $\frac{x e^{3x}}{3} + C$
- D. $\frac{x^2 e^{3x}}{6} + C$

[2 points] 3. $\int \frac{3x + 1}{x^2 - 1} dx =$

- A. $2 \ln |x - 1| + \ln |x + 1| + C$
- B. $\ln |x - 1| + 2 \ln |x + 1| + C$
- C. $\frac{3}{2} \ln |x^2 - 1| + C$
- D. $\frac{3}{2} \ln |x^2 - 1| + \arctan x + C$

[2 points] 4. $\int_2^{\infty} \frac{dx}{x(\ln x)^2}$ is

- A. $\ln 2$
- B. $\frac{1}{\ln 2}$
- C. $\frac{1}{2}$
- D. divergent

[2 points] 5. Let $y = f(x)$ be the solution of $\frac{dy}{dx} = x + y$ with $f(0) = 1$. Using Euler's method with two steps of size 0.5, the approximation to $f(1)$ is

- A. 1.5
- B. 2
- C. 2.5
- D. 3

[2 points] 6. A population $P(t)$ satisfies the logistic differential equation $\frac{dP}{dt} = 3P - 0.006P^2$ with $P(0) = 50$. The limiting value of P as $t \rightarrow \infty$ and the value of P at which P is growing fastest are, respectively,

- A. 500 and 250
- B. 500 and 500
- C. 1000 and 500
- D. 250 and 125

[2 points] 7. A curve is given parametrically by $x(t) = t^2$ and $y(t) = \frac{2}{3}t^3$. The length of the curve from $t = 0$ to $t = \sqrt{3}$ is

- A. $\frac{7}{3}$
- B. $\frac{14}{3}$
- C. $\frac{16}{3}$
- D. $\frac{28}{3}$

[2 points] 8. A particle moves in the xy -plane with velocity vector $\langle 2t, 3t^2 \rangle$ for $t \geq 0$. At time $t = 1$ the particle is at the point $(2, 5)$. Its position at time $t = 2$ is

- A. $(3, 7)$
- B. $(4, 8)$
- C. $(5, 12)$
- D. $(6, 13)$

[2 points] 9. The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 3^n}$ is

- A. $(-1, 5)$
- B. $[-1, 5)$
- C. $(-1, 5]$
- D. $[-1, 5]$

[2 points] 10. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \dots$ converges to a sum S . If the first four terms are used to approximate S , the alternating series error bound guarantees that the error is at most

- A. $\frac{1}{16}$
- B. $\frac{1}{25}$
- C. $\frac{1}{36}$
- D. $\frac{1}{5}$

[2 points] 11. A function f has derivatives of all orders, and $|f^{(4)}(x)| \leq 6$ for all x in $[0, 1]$. If the third-degree Taylor polynomial for f about $x = 0$ is used to approximate $f(\frac{1}{2})$, the Lagrange error bound guarantees that the error is at most

- A. $\frac{1}{16}$
- B. $\frac{1}{32}$
- C. $\frac{1}{64}$
- D. $\frac{3}{8}$

[2 points] 12. The third-degree Taylor polynomial for $f(x) = \ln x$ about $x = 1$ is

- A. $(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$
- B. $(x-1) - \frac{(x-1)^2}{2!} + \frac{(x-1)^3}{3!}$
- C. $(x-1) + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$
- D. $1 + (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$

Calculations (2 questions)

[9 points] 13. Series. Let f be the function defined by $f(x) = e^{-x^2}$, and let g be the function defined by $g(x) = \int_0^x f(t) dt$. You may use the fact that the Maclaurin series for e^u is $\sum_{n=0}^{\infty} \frac{u^n}{n!}$, which converges for all real u .

[3 points] (a) Write the first four nonzero terms and the general term of the Maclaurin series for f . Use the series to find $f^{(6)}(0)$.

[2 points] (b) Write the first four nonzero terms and the general term of the Maclaurin series for g .

[2 points] (c) Use the first two nonzero terms of the series in part (b) to approximate $g\left(\frac{1}{2}\right)$. Show that this approximation differs from the true value of $g\left(\frac{1}{2}\right)$ by less than $\frac{1}{300}$.

[2 points] (d) Find $\lim_{x \rightarrow 0} \frac{g(x) - x}{x^3}$.

[9 points] 14. Polar curves. For $0 \leq \theta \leq \pi$ the polar curve $r = 4 \sin \theta$ is the circle of radius 2 centered at the point $(0, 2)$, and the polar curve $r = 2$ is the circle of radius 2 centered at the origin. The two circles cross at two points above the x -axis. Let R be the region that lies inside the circle $r = 4 \sin \theta$ and outside the circle $r = 2$.

[2 points] (a) Find all values of θ in $[0, \pi]$ at which the two curves intersect.

[3 points] (b) Find the area of R .

[2 points] (c) Find the slope of the line tangent to the curve $r = 4 \sin \theta$ at the point where $\theta = \frac{\pi}{6}$.

[2 points] (d) A particle moves along the curve $r = 4 \sin \theta$ so that $\frac{d\theta}{dt} = 3$ for all t . Find the rate of change of the particle's distance from the origin with respect to time at the instant when $\theta = \frac{\pi}{3}$.