Name: ______________________

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AP Calculus AB — Practice Exam

> **Instructions:** Answer all questions in the spaces provided. Show your working — method marks are awarded even when the final answer is wrong. The answer key with worked solutions is available at examtex.com/free-exams.
>
> **Time Limit:** 60 minutes

Multiple Choice (12 questions)

[2 points] 1. Evaluate $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$.

A. 0
B. 3
C. 6
D. The limit does not exist

[2 points] 2. $\lim_{h \to 0} \dfrac{\sqrt{9 + h} - 3}{h}$ is:

A. 0
B. $\frac{1}{6}$
C. $\frac{1}{3}$
D. nonexistent

[2 points] 3. A function f is continuous on $[1, 4]$ with $f(1) = -2$ and $f(4) = 5$. Which of the following must be true?

A. f has a zero in the open interval $(1, 4)$
B. f is increasing on $[1, 4]$
C. $f'(c) = \frac{7}{3}$ for some c in $(1, 4)$
D. f has exactly one zero in $(1, 4)$

[2 points] 4. If $y = \sin(x^2)$, then $\dfrac{dy}{dx} =$

A. $\cos(x^2)$
B. $2x \cos(x^2)$
C. $2x \cos(2x)$
D. $-2x \cos(x^2)$

[2 points] 5. If $f(x) = x^2 e^x$, then $f'(x) =$

A. $2xe^x$
B. $x^2 e^x$
C. $e^x(x^2 + 2x)$
D. $e^x(x^2 - 2x)$

[2 points] 6. The curve $x^2 + y^2 = 25$ passes through the point $(3, 4)$. The slope of the curve at that point is:

A. $\frac{3}{4}$
B. $-\frac{3}{4}$
C. $\frac{4}{3}$
D. $-\frac{4}{3}$

[2 points] 7. The radius of a sphere is increasing at 2 centimeters per second. In terms of π, how fast is the volume $V = \frac{4}{3}\pi r^3$ increasing when the radius is 3 centimeters?

A. 24π cubic centimeters per second
B. 36π cubic centimeters per second
C. 72π cubic centimeters per second
D. 144π cubic centimeters per second

[2 points] 8. For $f(x) = x^2$ on the interval $[1, 3]$, the value of c guaranteed by the Mean Value Theorem satisfies $f'(c) =$ the average rate of change. Then $c =$

A. $\frac{3}{2}$
B. 2
C. $\sqrt{5}$
D. $\frac{5}{2}$

[2 points] 9. $\displaystyle\int 2x\,(x^2 + 1)^3\,dx =$

A. $(x^2 + 1)^4 + C$
B. $\frac{(x^2+1)^4}{4} + C$
C. $2(x^2 + 1)^4 + C$
D. $x^2(x^2 + 1)^3 + C$

[2 points] 10. If $g(x) = \displaystyle\int_0^x \sqrt{1 + t^3}\,dt$, then $g'(2) =$

A. 3
B. 9
C. $\sqrt{5}$
D. 0

[2 points] 11. Given $\int_0^5 f(x)\,dx = 10$ and $\int_0^2 f(x)\,dx = 4$, find $\int_2^5 f(x)\,dx$.

A. 6
B. 14
C. -6
D. 40

[2 points] 12. The average value of $f(x) = 3x^2$ on the interval $[0, 2]$ is:

A. 4
B. 8
C. 12
D. 6

Calculations (3 questions)

[9 points] 13. Particle motion. A particle moves along the x-axis so that its position at time t is $x(t) = t^3 - 6t^2 + 9t$ for $0 \le t \le 5$, where x is measured in meters and t in seconds.

$$x(t) = t^3 - 6t^2 + 9t, \qquad 0 \le t \le 5$$

[2 points] (a) Find the velocity $v(t)$ and acceleration $a(t)$ of the particle.

[3 points] (b) On what interval(s) is the particle moving to the left? Justify your answer.

[2 points] (c) At time $t = 2.5$, is the speed of the particle increasing or decreasing? Justify your answer.

[2 points] (d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

[9 points] 14. Area and volume. Let R be the region in the first quadrant bounded by the graphs of $y = 4x - x^2$ and $y = x$.

$y = 4x - x^2, \qquad y = x$

[3 points] (a) Find the points of intersection and the area of R.

[4 points] (b) The region R is the base of a solid whose cross-sections perpendicular to the x-axis are squares. Find the volume of the solid.

[2 points] (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the x-axis.

[9 points] 15. Accumulation. Water flows into a tank at the rate $R(t) = 20 - t^2$ liters per minute and drains out at a constant 11 liters per minute, for $0 \leq t \leq 4$. At time $t = 0$ the tank contains 30 liters.

$R(t) = 20 - t^2, \qquad D(t) = 11, \qquad 0 \leq t \leq 4$

[2 points] (a) How much water flows INTO the tank during the 4 minutes?

[3 points] (b) How much water is in the tank at time $t = 4$?

[2 points] (c) At time $t = 2$, is the amount of water in the tank increasing or decreasing? Justify.

[2 points] (d) At what time t in $[0, 4]$ is the amount of water in the tank greatest? Justify, and find that maximum amount.

Answer Key

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1. **C**

2. **B**

3. **A**

4. **B**

5. **C**

6. **B**

7. **C**

8. **B**

9. **B**

10. **A**

11. **A**

12. **A**

13. (a) $v(t) = x'(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3)$ meters per second, and $a(t) = v'(t) = 6t - 12$ meters per second squared.

(b) The particle moves left when $v(t) < 0$. Since $v(t) = 3(t-1)(t-3)$, the sign chart gives $v > 0$ on $(0,1)$, $v < 0$ on $(1,3)$, and $v > 0$ on $(3,5)$. The particle moves to the left on $(1,3)$ because the velocity is negative there.

(c) $v(2.5) = 3(1.5)(-0.5) = -2.25 < 0$ and $a(2.5) = 6(2.5) - 12 = 3 > 0$. Velocity and acceleration have opposite signs, so the speed is decreasing at $t = 2.5$. (Same sign $\rightarrow$ speeding up; opposite signs $\rightarrow$ slowing down.)

(d) The particle changes direction at $t = 1$ (where $v = 0$). Positions: $x(0) = 0$, $x(1) = 1-6+9 = 4$, $x(2) = 8 - 24 + 18 = 2$. Total distance $= |4-0| + |2-4| = 4 + 2 = 6$ meters. (Displacement is only 2 meters — total distance must count the reversal.)

14. (a) Intersections: $4x - x^2 = x \Rightarrow 3x - x^2 = 0 \Rightarrow x(3-x) = 0$, so $x = 0$ and $x = 3$ (points $(0,0)$ and $(3,3)$). On $(0,3)$ the parabola lies above the line, so Area $= \int_0^3 (3x - x^2)\,dx = \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 = \frac{27}{2} - 9 = \frac{9}{2}$.

(b) The side of each square is the vertical gap $s(x) = 3x - x^2$, so $V = \int_0^3 (3x - x^2)^2\,dx = \int_0^3 (9x^2 - 6x^3 + x^4)\,dx = \left[3x^3 - \frac{3x^4}{2} + \frac{x^5}{5}\right]_0^3 = 81 - \frac{243}{2} + \frac{243}{5} = \frac{81}{10}$.

(c) Washer method — outer radius $4x - x^2$, inner radius x: $V = \pi \int_0^3 \left[(4x - x^2)^2 - x^2\right] dx$. (Setup earns the points; the outer/inner order is what graders check.)

15. (a) $\int_0^4 (20 - t^2)\,dt = \left[20t - \frac{t^3}{3}\right]_0^4 = 80 - \frac{64}{3} = \frac{176}{3} \approx 58.7$ liters.

(b) Amount $= 30 + \int_0^4 (R(t) - 11)\,dt = 30 + \int_0^4 (9 - t^2)\,dt = 30 + \left[9t - \frac{t^3}{3}\right]_0^4 = 30 + 36 - \frac{64}{3} =$

$\frac{134}{3} \approx 44.7$ liters.

(c) The net rate is $R(2) - 11 = 20 - 4 - 11 = 5 > 0$ liters per minute, so the amount of water is increasing at $t = 2$.

(d) The net rate $9-t^2$ is zero at $t = 3$, positive before and negative after — so by the First Derivative Test the amount is maximal at $t = 3$. Maximum $= 30 + \int_0^3 (9 - t^2)\, dt = 30 + 27 - 9 = 48$ liters.

Worked Solutions

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