

Name: _____

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AP Calculus AB — Practice Exam

Instructions: Answer all questions in the spaces provided. Show your working — method marks are awarded even when the final answer is wrong. The answer key with worked solutions is available at examtex.com/free-exams.

Time Limit: 60 minutes

Multiple Choice (12 questions)

[2 points] 1. Evaluate $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$.

- A. 0
- B. 3
- C. 6
- D. The limit does not exist

[2 points] 2. $\lim_{h \rightarrow 0} \frac{\sqrt{9+h} - 3}{h}$ is:

- A. 0
- B. $\frac{1}{6}$
- C. $\frac{1}{3}$
- D. nonexistent

[2 points] 3. A function f is continuous on $[1, 4]$ with $f(1) = -2$ and $f(4) = 5$. Which of the following must be true?

- A. f has a zero in the open interval $(1, 4)$
- B. f is increasing on $[1, 4]$
- C. $f'(c) = \frac{7}{3}$ for some c in $(1, 4)$
- D. f has exactly one zero in $(1, 4)$

[2 points] 4. If $y = \sin(x^2)$, then $\frac{dy}{dx} =$

- A. $\cos(x^2)$
- B. $2x \cos(x^2)$
- C. $2x \cos(2x)$
- D. $-2x \cos(x^2)$

[2 points] 5. If $f(x) = x^2 e^x$, then $f'(x) =$

- A. $2x e^x$
- B. $x^2 e^x$
- C. $e^x(x^2 + 2x)$
- D. $e^x(x^2 - 2x)$

[2 points] 6. The curve $x^2 + y^2 = 25$ passes through the point $(3, 4)$. The slope of the curve at that point is:

- A. $\frac{3}{4}$
- B. $-\frac{3}{4}$
- C. $\frac{4}{3}$
- D. $-\frac{4}{3}$

[2 points] 7. The radius of a sphere is increasing at 2 centimeters per second. In terms of π , how fast is the volume $V = \frac{4}{3}\pi r^3$ increasing when the radius is 3 centimeters?

- A. 24π cubic centimeters per second
- B. 36π cubic centimeters per second
- C. 72π cubic centimeters per second
- D. 144π cubic centimeters per second

[2 points] 8. For $f(x) = x^2$ on the interval $[1, 3]$, the value of c guaranteed by the Mean Value Theorem satisfies $f'(c) =$ the average rate of change. Then $c =$

- A. $\frac{3}{2}$
- B. 2
- C. $\sqrt{5}$
- D. $\frac{5}{2}$

[2 points] 9. $\int 2x(x^2 + 1)^3 dx =$

- A. $(x^2 + 1)^4 + C$
- B. $\frac{(x^2+1)^4}{4} + C$
- C. $2(x^2 + 1)^4 + C$
- D. $x^2(x^2 + 1)^3 + C$

[2 points] 10. If $g(x) = \int_0^x \sqrt{1+t^3} dt$, then $g'(2) =$

- A. 3
- B. 9
- C. $\sqrt{5}$
- D. 0

[2 points] 11. Given $\int_0^5 f(x) dx = 10$ and $\int_0^2 f(x) dx = 4$, find $\int_2^5 f(x) dx$.

- A. 6
- B. 14
- C. -6
- D. 40

[2 points] 12. The average value of $f(x) = 3x^2$ on the interval $[0, 2]$ is:

- A. 4
- B. 8
- C. 12
- D. 6

Calculations (3 questions)

[9 points] 13. Particle motion. A particle moves along the x-axis so that its position at time t is $x(t) = t^3 - 6t^2 + 9t$ for $0 \leq t \leq 5$, where x is measured in meters and t in seconds.

$$x(t) = t^3 - 6t^2 + 9t, \quad 0 \leq t \leq 5$$

[2 points] (a) Find the velocity $v(t)$ and acceleration $a(t)$ of the particle.

[3 points] (b) On what interval(s) is the particle moving to the left? Justify your answer.

[2 points] (c) At time $t = 2.5$, is the speed of the particle increasing or decreasing? Justify your answer.

[2 points] (d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

[9 points] 14. Area and volume. Let R be the region in the first quadrant bounded by the graphs of $y = 4x - x^2$ and $y = x$.

$$y = 4x - x^2, \quad y = x$$

[3 points] (a) Find the points of intersection and the area of R .

- [4 points]** (b) The region R is the base of a solid whose cross-sections perpendicular to the x -axis are squares. Find the volume of the solid.

- [2 points]** (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the x -axis.

- [9 points]** 15. Accumulation. Water flows into a tank at the rate $R(t) = 20 - t^2$ liters per minute and drains out at a constant 11 liters per minute, for $0 \leq t \leq 4$. At time $t = 0$ the tank contains 30 liters.

$$R(t) = 20 - t^2, \quad D(t) = 11, \quad 0 \leq t \leq 4$$

- [2 points]** (a) How much water flows INTO the tank during the 4 minutes?

[3 points] (b) How much water is in the tank at time $t = 4$?

[2 points] (c) At time $t = 2$, is the amount of water in the tank increasing or decreasing? Justify.

[2 points] (d) At what time t in $[0, 4]$ is the amount of water in the tank greatest? Justify, and find that maximum amount.