

Name: _____

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AP Precalculus — Practice Exam

Instructions: Section I (multiple choice): no calculator. Section II (free response): a graphing calculator is permitted. Show all working and round final answers to three decimal places unless the question says otherwise.

Time Limit: 60 minutes

Multiple Choice (14 questions)

[2 points] 1. Let $p(x) = (3 - x)(x + 1)^2(2x - 5)$. Which of the following describes the end behavior of p ?

- A. $p(x) \rightarrow \infty$ as $x \rightarrow \infty$ and $p(x) \rightarrow \infty$ as $x \rightarrow -\infty$
- B. $p(x) \rightarrow -\infty$ as $x \rightarrow \infty$ and $p(x) \rightarrow \infty$ as $x \rightarrow -\infty$
- C. $p(x) \rightarrow \infty$ as $x \rightarrow \infty$ and $p(x) \rightarrow -\infty$ as $x \rightarrow -\infty$
- D. $p(x) \rightarrow -\infty$ as $x \rightarrow \infty$ and $p(x) \rightarrow -\infty$ as $x \rightarrow -\infty$

[2 points] 2. Let $p(x) = (x + 2)^2(x - 1)^3(x - 4)$. Which statement about the graph of p is true?

- A. The graph crosses the x -axis at $x = -2$, $x = 1$ and $x = 4$.
- B. The graph touches the x -axis without crossing at $x = -2$ and $x = 1$ and crosses at $x = 4$.
- C. The graph touches the x -axis without crossing at $x = -2$ and crosses at $x = 1$ and $x = 4$.
- D. The graph crosses the x -axis at $x = -2$ and touches without crossing at $x = 1$ and $x = 4$.

[2 points] 3. Let $r(x) = \frac{2x^2 - 8}{x^2 - 5x + 6}$. Which of the following is true about the graph of r ?

- A. Vertical asymptotes at $x = 2$ and $x = 3$; horizontal asymptote $y = 2$
- B. A hole at $x = 2$, a vertical asymptote at $x = 3$ and a horizontal asymptote $y = 0$
- C. A hole at $x = 2$, a vertical asymptote at $x = 3$ and a horizontal asymptote $y = 2$
- D. Vertical asymptotes at $x = 2$ and $x = 3$ and no horizontal asymptote

[2 points] 4. As $x \rightarrow \infty$, the graph of $f(x) = \frac{x^2 + 3x - 1}{x - 1}$ approaches the line

- A. $y = x$
- B. $y = x + 2$
- C. $y = x + 3$
- D. $y = x + 4$

- [2 points]** 5. A function f has $f(1) = 3$, $f(2) = 7$, $f(3) = 10$ and $f(4) = 12$. Based on these values, which best describes f on the interval $1 \leq x \leq 4$?
- Increasing and concave up
 - Decreasing and concave down
 - Increasing and concave down
 - Linear with slope 3
- [2 points]** 6. An exponential function $f(x) = a \cdot b^x$ satisfies $f(2) = 12$ and $f(5) = 96$. The value of $f(0)$ is
- 1.5
 - 3
 - 4
 - 6
- [2 points]** 7. If $\log_b 2 = p$ and $\log_b 3 = q$, then $\log_b \frac{18}{b} =$
- $p + 2q - 1$
 - $2p + q - 1$
 - $p + 2q$
 - $p + 2q - b$
- [2 points]** 8. A set of data points (x, y) is plotted on a semi-log graph with a logarithmic (base 10) scale on the vertical axis and a linear scale on the horizontal axis. The points lie on a straight line with slope 0.5 and vertical intercept 2. Which function models y in terms of x ?
- $y = 2 + 0.5x$
 - $y = 10^{2x+0.5}$
 - $y = 100 \cdot x^{0.5}$
 - $y = 100 \cdot 10^{0.5x}$
- [2 points]** 9. A sample of a radioactive substance decays exponentially. Its mass falls from 80 milligrams to 10 milligrams in 15 days. The half-life of the substance is
- 3 days
 - 5 days
 - 7.5 days
 - $\frac{15}{8}$ days
- [2 points]** 10. If $f(x) = 3 + \ln(x - 2)$ for $x > 2$, then the inverse function is $f^{-1}(x) =$
- $e^{x-3} + 2$
 - $e^{x-2} + 3$
 - $e^{x+2} - 3$
 - $\ln(x - 3) + 2$

[2 points] 11. If $\sin \theta = \frac{3}{5}$ and $\frac{\pi}{2} < \theta < \pi$, then $\sin 2\theta =$

- A. $\frac{24}{25}$
- B. $-\frac{24}{25}$
- C. $\frac{7}{25}$
- D. $-\frac{7}{25}$

[2 points] 12. At a pier the water depth varies sinusoidally with time. The depth is at its minimum of 2 meters at 3 a.m. and reaches its next maximum of 8 meters at 9 a.m. Which function gives the depth $d(t)$ in meters, where t is the number of hours after midnight?

- A. $d(t) = 5 - 3 \cos\left(\frac{\pi}{6}(t - 3)\right)$
- B. $d(t) = 5 + 3 \cos\left(\frac{\pi}{6}(t - 3)\right)$
- C. $d(t) = 5 - 3 \cos\left(\frac{\pi}{12}(t - 3)\right)$
- D. $d(t) = 5 - 3 \sin\left(\frac{\pi}{6}(t - 3)\right)$

[2 points] 13. How many solutions does the equation $2 \sin^2 \theta - \sin \theta - 1 = 0$ have on the interval $0 \leq \theta < 2\pi$?

- A. 1
- B. 2
- C. 3
- D. 4

[2 points] 14. The polar function $r = 3 + 3 \cos \theta$ is graphed for $0 \leq \theta \leq 2\pi$. On which interval of θ is the distance between the point on the curve and the origin decreasing as θ increases?

- A. $0 < \theta < \pi$
- B. $\pi < \theta < 2\pi$
- C. $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$
- D. $0 < \theta < \frac{\pi}{2}$ and $\frac{3\pi}{2} < \theta < 2\pi$

Calculations (2 questions)

[8 points] 15. Modelling a data set. A company launched a mobile app and recorded the number of active subscribers, in thousands, at the end of each of its first six months. At the end of month 1 it had 4.2 thousand subscribers; month 2, 7.2 thousand; month 3, 12.3 thousand; month 4, 20.9 thousand; month 5, 35.5 thousand; and month 6, 60.3 thousand. Let $S(t)$ be the number of subscribers, in thousands, at the end of month t .

- [2 points]** (a) Find the average rate of change of S over the interval $1 \leq t \leq 3$ and over the interval $4 \leq t \leq 6$. Include units, and explain what the comparison tells you about the shape of the graph of S .

- [1 point]** (b) Explain, using the data, why an exponential model is more appropriate than a linear model for S .

- [2 points]** (c) Use the data for months 1 and 6 to write an exponential model of the form $S(t) = a \cdot b^t$. Round a and b to three decimal places.

[2 points] (d) Use your model to predict the number of subscribers at the end of month 9. Then give one reason why this prediction may be unreliable.

[1 point] (e) According to your model, during which month does the company first reach 150 thousand subscribers?

[8 points] 16. Modelling a periodic context. A Ferris wheel has a diameter of 40 meters, and its lowest point is 2 meters above the ground. The wheel turns at a constant rate and completes one full rotation every 4 minutes. A rider boards at the lowest point at time $t = 0$ and the wheel starts turning immediately. Let $h(t)$ be the height of the rider above the ground, in meters, t minutes after boarding.

[2 points] (a) State the midline, the amplitude and the period of h , and explain what each represents in the context of the Ferris wheel.

[2 points] (b) Write a function $h(t)$ that models the height of the rider.

[1 point] (c) Find the height of the rider 20 seconds after boarding.

[2 points] (d) Find the first two times after boarding at which the rider is exactly 32 meters above the ground.

- [1 point]** (e) On the interval $0 < t < 4$, find the interval of time during which the height of the rider is increasing at a decreasing rate. Explain your reasoning.