

Name: _____

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IB Mathematics AI SL — Practice Exam

Instructions: A graphic display calculator (GDC) is required for this paper. Answer all questions. Unless otherwise stated in the question, give answers exactly or correct to three significant figures. Show your working: method marks are awarded even when the final answer is wrong.

Time Limit: 90 minutes

- [6 points] 1. Maya invests 8000 USD in a savings account that pays a nominal annual interest rate of 4.2%, compounded monthly. She leaves the money in the account for 5 years. Maya also borrows 15 000 USD to buy a car. The loan charges a nominal annual interest rate of 6%, compounded monthly, and is repaid in equal monthly instalments over 4 years.
- [2 points] (a) Calculate the value of Maya's investment at the end of the 5 years. Give your answer correct to two decimal places.
- [2 points] (b) Tom estimates the value of Maya's investment by assuming simple interest of 4.2% per year for 5 years. Calculate the percentage error in Tom's estimate.

[2 points] (c) Find the monthly repayment on the car loan, and hence the total interest Maya pays.

[7 points] 2. Maya and Tom each start a savings plan on 1 January. Maya saves 40 USD in the first month and increases the amount she saves by 5 USD every month. Tom saves 40 USD in the first month and increases the amount he saves by 8% every month.

[2 points] (a) Find the amount Maya saves in the twelfth month.

[2 points] (b) Find the total amount Maya saves in the first two years.

[2 points] (c) Find the total amount Tom saves in the first two years.

[1 point] (d) State who saves more over the two years, and by how much.

[7 points] 3. A triangular field ABC has $AB = 85$ m, $AC = 62$ m and angle $BAC = 48$ degrees. A vertical flagpole of height 12 m stands at C.

[3 points] (a) Find the length of BC.

[2 points] (b) Find the area of the field.

[2 points] (c) Find the angle of elevation of the top of the flagpole from B.

[6 points] 4. A town has three mobile-phone masts at $A(0, 0)$, $B(8, 0)$ and $C(4, 6)$, where the coordinates are in kilometres. Every phone connects to the nearest mast, so the town is divided into three regions by a Voronoi diagram with sites A, B and C.

[1 point] (a) Write down the equation of the perpendicular bisector of AB.

[3 points] (b) Find the equation of the perpendicular bisector of AC. Give your answer in the form $ax + by = c$, where a , b and c are integers.

[1 point] (c) Hence find the coordinates of the vertex of the Voronoi diagram, the point that is equidistant from all three masts.

[1 point] (d) A new house is built at $(5, 2)$. Determine which mast the phones in this house will connect to.

[6 points] 5. A grain silo consists of a cylinder of radius 3 m and height 8 m with a cone of height 4 m on top. The cone has the same radius as the cylinder. The silo stands on flat ground.

[3 points] (a) Calculate the volume of the silo.

[1 point] (b) Show that the slant height of the cone is 5 m.

[2 points] (c) The outside of the silo, excluding the base on the ground, is to be painted. Calculate the area to be painted.

[6 points] 6. The times, in minutes, taken by 12 students to complete a puzzle were 14, 18, 22, 23, 25, 25, 27, 29, 31, 34, 38 and 52.

[1 point] (a) Write down the median time.

[2 points] (b) Find the interquartile range.

[2 points] (c) Show that the time of 52 minutes is an outlier.

[1 point] (d) Find the mean time.

[6 points] 7. The masses of the apples grown on a farm are normally distributed with mean 165 g and standard deviation 12 g.

[2 points] (a) Find the probability that a randomly chosen apple has a mass greater than 180 g.

[2 points] (b) The heaviest 5% of the apples are sold as premium apples. Find the minimum mass of a premium apple.

[2 points] (c) Six apples are chosen at random. Find the probability that at least one of them has a mass greater than 180 g.

[6 points] 8. A factory makes light bulbs on two machines. Machine A makes 60% of the bulbs, and 3% of its bulbs are faulty. Machine B makes the remaining 40% of the bulbs, and 5% of its bulbs are faulty. A bulb is chosen at random from the factory's output.

[2 points] (a) Find the probability that the bulb is faulty.

[2 points] (b) Given that the bulb is faulty, find the probability that it was made by machine B.

[2 points] (c) Two bulbs are chosen at random and independently. Find the probability that exactly one of them is faulty.

[10 points] 9. A teacher records the number of hours, x , that each of eight students spent revising for a test, and their scores, y , out of 100. The pairs (x, y) are $(1, 46)$, $(2, 50)$, $(3, 58)$, $(4, 57)$, $(5, 66)$, $(6, 70)$, $(7, 71)$ and $(8, 82)$. The teacher then surveys 150 students across the school about their main revision method and whether they passed the test. Of the 50 students who mainly re-read their notes, 28 passed; of the 40 who used flashcards, 26 passed; of the 60 who did practice tests, 48 passed.

[2 points] (a) Find Pearson's product-moment correlation coefficient, r , and describe the correlation.

[2 points] (b) Write down the equation of the regression line of y on x .

[1 point] (c) Use your regression line to estimate the score of a student who revised for 5.5 hours.

[1 point] (d) The teacher carries out a chi-square test of independence at the 5% significance level. Write down the null hypothesis.

[1 point] (e) Show that the expected number of students who did practice tests and passed is 40.8.

[2 points] (f) Find the chi-square statistic and the p-value for the test.

[1 point] (g) State, with a reason, the conclusion of the test.

[10 points] 10. A garden designer models the width of a flower bed. One edge of the bed is a straight path along the x-axis, and the other edge follows the curve $y = w(x)$, where $w(x) = 0.1x^3 - 0.9x^2 + 2.4x$ for $0 \leq x \leq 5$, with x and $w(x)$ in metres. The bed ends with a straight edge at $x = 5$.

$$w(x) = 0.1x^3 - 0.9x^2 + 2.4x, \quad 0 \leq x \leq 5$$

[2 points] (a) Find $w'(x)$.

[4 points] (b) Find the coordinates of the local maximum point and the local minimum point of the curve.

[3 points] (c) Use the trapezoidal rule with five intervals of width 1 m to estimate the area of the flower bed.

[1 point] (d) Use your GDC to find the actual area of the bed, and state whether the trapezoidal rule gives an underestimate or an overestimate.