Name: _____

IB Mathematics: Analysis and Approaches SL — Practice Exam

Instructions: Paper 1 style — NO CALCULATOR permitted. Answer all questions in the spaces provided. Show your working: method marks are awarded even when the final answer is wrong. Give exact answers (fractions, surds, multiples of pi) unless the question says otherwise.

Time Limit: 75 minutes

[6 points] 1. An arithmetic sequence has third term $u_3 = 11$ and seventh term $u_7 = 23$.

[4 points] (a) Find the first term u_1 and the common difference d.

[2 points] (b) Find S_{20}, the sum of the first 20 terms.

[5 points] 2. The functions f and g are defined by $f(x) = 2x - 3$ for $x \in \mathbb{R}$, and $g(x) = \frac{x+1}{x-2}$ for $x \neq 2$.

$f(x) = 2x - 3, \qquad g(x) = \frac{x+1}{x-2}, \quad x \neq 2$

[2 points] (a) Find $(g \circ f)(3)$.

[3 points] (b) Find $g^{-1}(x)$, stating its domain.

[6 points] 3. This question is about exponents and logarithms.

[3 points] (a) Solve $3^{2x-1} = 27^{x-2}$.

[3 points] (b) Show that $2\log_3 6 - \log_3 4 = 2$.

[5 points] 4. This question is about trigonometry with exact values.

[3 points] (a) Solve $2\cos x = \sqrt{3}$ for $0 \leq x \leq 2\pi$.

[2 points] (b) In triangle ABC, $AB = 6$ cm, $AC = 8$ cm and $B\hat{A}C = \frac{\pi}{3}$. Find the exact area of the triangle.

[6 points] 5. The curve $y = f(x)$ where $f(x) = x^3 - 3x + 1$ passes through the point $P(2, 3)$.

$f(x) = x^3 - 3x + 1$

[2 points] (a) Find $f'(x)$.

[2 points] (b) Find the equation of the tangent to the curve at P.

[2 points] (c) Find the equation of the normal to the curve at P.

[6 points] 6. The probability that it rains on a given morning is $\frac{1}{4}$. If it rains, the probability that Mia cycles to school is $\frac{1}{5}$; if it does not rain, the probability that she cycles is $\frac{3}{5}$. Let R be the event "it rains" and C the event "Mia cycles". A tree diagram may help.

[2 points] (a) Find $P(C)$, the probability that Mia cycles to school.

[2 points] (b) Given that Mia cycles to school, find the probability that it rained.

[2 points] (c) Determine, with a reason, whether the events R and C are independent.

[14 points] 7. The function f is defined by $f(x) = x^3 - 6x^2 + 9x$ for $x \in \mathbb{R}$. The curve $y = f(x)$ is studied throughout this question.

$f(x) = x^3 - 6x^2 + 9x$

[4 points] (a) Find $f'(x)$, and hence find the coordinates of the two stationary points of the curve.

[2 points] (b) Justify the nature of each stationary point.

[2 points] (c) Find $f''(x)$, and hence find the coordinates of the point of inflexion.

[4 points] (d) Find the exact area of the region enclosed by the curve and the x-axis.

[2 points] (e) Find the equation of the tangent to the curve at the point of inflexion.